Modelling Nigerian Banks' Share Prices Using Smooth Transition GARCH Models

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This paper examined the application of nonlinear Smooth Transition-Generalized Autoregressive Conditional Heteroscedasticity (ST-GARCH) model of Hagerud on prices of banks' shares in Nigeria. The methodology is informed by the failure of the conventional GARCH model to capture the asymmetric properties of the banks' daily share prices. The asymmetry and non-linearity in the model dynamics make it useful for generating nonlinear conditional variance series. From the empirical analysis, we obtained the conditional volatility of each bank's share price return. The highest volatility persistence was observed in Bank 6, while Bank 12 had the least volatility. Evidently, about 25% of the investigated banks exhibited linear volatility behaviour, while the remaining banks showed nonlinear volatility specifications. Given the level of risk associated with investment in stocks, investors and financial analysts could consider volatility modelling of bank share prices with variants of the ST-GARCH models. The impact of news is an important feature that relevant agencies could study so as to be guided while addressing underlying issues in the banking system.

Keywords: Specification, Smooth Transition-GARCH, Banks Stocks, Nigeria stock exchange

JEL Classification: C22

1.0 Introduction

Real world problems do not always satisfy the assumptions of linearity and/or stationarity, although time series econometric modelling is most often characterised by nonstationary and nonlinear models. These models are useful for understanding the behaviour of different time series in order to enhance prediction and forecasts. The dynamic nature of time series analysis therefore informs the need for further development(s) of the existing theories and also necessitates appropriate application of nonlinear models.

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In general, nonlinear time series exhibits characteristics such as cycles, asymmetries, bursts, jumps, chaos, thresholds, outliers, heteroscedasticity and/or mixtures of these components. It has received a growing interest from both theoretical and applied researchers and widely applicable in various forms, which include the Bilinear (BL), Markov Switching (MS), Threshold Autoregressive (TAR), Exponential Autoregressive (EAR), Smooth Transition Autoregressive (STAR) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) models.

The dynamics of economic and financial time series are often nonlinear, and recently, nonlinear modelling approaches are being used to capture the dynamics. Nonlinear models are found to perform better than the corresponding linear models. These nonlinear models are often applied to the level series (the original un-transformed series) except in cases where the interest is on studying the volatility in a series, which is in the form of heteroscedasticity. The series needs to be transformed first in order to obtain a new series, which reveals the inherent volatility more vividly.

Hagerud (1996) and Gonzalez-Rivera (1998), based on the initial work of Teräsvirta (1994)[4] proposed simultaneously, nonlinear GARCH types models for capturing asymmetric and symmetric nonlinearity of conditional variance series. These are the Smooth Transition-Autoregressive Conditional Heteroscedasticity (ST-ARCH) and the generalized version, ST-GARCH models, respectively. Following the idea of the Smooth Transition Autoregressive (STAR) model of Teräsvirta (1994) which classifies the financial market dynamics into two regimes of ups and downs, that is, the bull and bear states, when interest is in studying nonlinearity of volatility in the market structure using GARCH model, the series is transformed instead of applying the level series, and the most appropriate model is the ST-(G)ARCH model.

Like the STAR model, the ST-(G)ARCH is also of two forms: the Logistic Smooth Transition-GARCH (LST-(G)ARCH) and Exponential Smooth Transition-(G)ARCH (EST-(G)ARCH) models, for asymmetric and symmetric nonlinear volatility adjustments respectively. One important feature of the LST-(G)ARCH model is that it places the asymmetric effect of unexpected shocks (returns) on the conditional volatility. The conditional

[4]Teräsvirta (1994) proposed Smooth Transition Autoregressive (STAR) model, which is of two forms: the Logistic STAR (LSTAR) model for capturing asymmetric nonlinearity, and the Exponential STAR (ESTAR) model for capturing symmetric nonlinearity.

variance of the LST-(G)ARCH model possesses dynamics similar to those of the GJR-GARCH model in that, the parameter for the squared residual have one value when the residual is positive and another value when the residual is negative. Contrary to this, the EST-(G)ARCH model however allows the dynamics of the conditional variance to be independent of the signs of the past values of the returns.

Interestingly, both the LST-(G)ARCH and EST-(G)ARCH models allow studying the size of the effect of shocks, that is small and big shocks having separate effects, respectively. These models, therefore perform better than the GARCH model by allowing for both asymmetric and symmetric regime changes on the conditional volatility as a result of gradual change on the transition parameter which actually causes the nonlinearity. High frequency series such as stock returns are characterized with some stylized facts, among which are volatility clustering, fat-tail and asymmetry. Thus, the traditional assumption of normality in volatility modelling of financial time series could weaken the robustness of parameter estimates.

Extensive study has been done on volatility modelling in developed countries but less concern is given to the subject matter in Sub-Saharan Africa. In Nigeria, a number of studies have been carried out on modelling financial data and stock market volatility using GARCH model but only a few studies have been carried out on asymmetric volatility in stock prices, especially banks stocks. However, due to the inadequacy of the GARCH model to effectively capture nonlinearity and asymmetric properties of financial data, non-linear GARCH type models have been proposed to capture the regime switching behaviour.

Moreover, the smooth transition is an extension of the regime switching model that allows intermediate states or regimes. The idea of smooth transition was proposed to allow a more gradual change for the transition parameter (Hagerud, 1997). This model also provides more flexibility in the transition mechanism of the conditional volatility. Unlike the traditional threshold models that allow only two volatility regimes (a low volatility regime and a high volatility regime), the ST-GARCH gives room for intermediate regimes and allows the introduction of a smoother transition mechanism in the GARCH specification (Bonilla et al., 2006). In light of this, the ST-GARCH model will allow to highlight significant volatility characteristics of banks' share prices in this study.

To our knowledge, available literatures for the Nigerian case tend to capture nonlinear and asymmetric properties of financial data in one regime. In a volatile market like Nigeria, investors, policy makers and banks are certainly interested in the nature of stocks because it is proximity for the value of risk they incurred. In this regard, this paper aim to model banks share prices using the nonlinear Smooth Transition-Generalized Autoregressive Conditional Heteroscedasticity (hereafter, referred to as ST-GARCH) model using its variants LST-(G)ARCH and EST-(G)ARCH models, to capture nonlinear, asymmetric and symmetric properties of Nigerian banks stocks and also to determine the volatility behaviour (linear or nonlinear) of each bank.

This paper is further structured as follows; in the second section, a critical review of existing literature is presented, showing the gaps in volatility modelling in Nigeria. The third section explicitly discusses the methodology behind volatility modelling, and presents the data, its transformation and test procedure. The fourth section covers the discussion of empirical results, while section five concludes with some policy implications.

2.0 Review of Literature

Instability in stock prices major component is exhibited by the varying conditional variance (volatility) of the stock prices. What obviously interest investors in the stock markets are volatility nature of stock prices because high volatility could mean huge losses or gains and hence greater uncertainty. This makes it difficult for companies to raise capital in volatile markets. Shittu, Yaya and Oguntade (2009) examined the presence and or otherwise of volatility in the return on stock of the banking sector of the Nigerian stock market using the ARCH and GARCH models. The stock data of five major banks in Nigeria showed varying degrees of persistence in volatility with the return on Union Bank assets indicating weak evidence of volatility which implied that the stock of Union bank was relatively stable. Their results also showed that the volatility of stocks in the banking sector had strong influence on the other stocks in the Nigerian stock exchange.

Olowe (2009) found, amongst others, evidence of volatility persistence and leverage effects. His results showed that the stock market crash of 2008 was found to have contributed to the high volatility persistence in the Nigerian stock market, especially during the global financial crisis period. However, Okpara and Nwezeaku (2009) randomly selected forty one (41) companies from the Nigerian Stock Exchange to examine the effect of the idiosyncratic risk and beta risk on returns using data from 1996 to 2005. By applying

EGARCH (1, 3) model, their result showed less volatility persistence and established the existence of leverage effect in the Nigeria stock market, implying that bad news drives volatility more than good news.

Dallah and Ade (2010) examined the volatility of daily stock returns of Nigerian Insurance Stocks using twenty six (26) insurance companies. Their empirical results revealed that the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) was more suitable in modelling volatility of stock price returns as it out performed the other models in model-estimation and out-of-sample volatility forecasting. While few writers believed that certain price trends and patterns exist to enable the investors to make better predictions of the expected values of future change in stock market price, majority of these studies concluded that past price data alone cannot form the basis for predicting the expected values of price movements in the stock market (Eriki and Idolor, 2010).

Eriki and Idolor (2010) employed Markovian Analysis to establish the behaviour of stock prices in the Nigerian capital market by examining eight stocks, randomly selected from the banking sector for the period of January 2005 to June 2008. Their result showed that stock prices were random. They also argued that different companies were affected at different times by new information that could produce significant differences in the runs and reversal patterns among daily stock prices.

Abdalla and Winker (2012) examined stock market volatility in two African exchanges; the Khartoum stock exchange (from Sudan) and the Cairo and Alexandria stock exchange (from Egypt) using daily closing prices on general indices in the two markets. Different univariate specifications of the GARCH model were employed and their results provided evidence of positive correlation between volatility and the expected stock returns. Furthermore, the asymmetric GARCH models find a significant evidence for asymmetry in the stock returns of the two markets, confirming the presence of leverage effect in the return series.

For capturing nonlinearities and structural breaks in economic variables in two regimes, Smooth Transition Regression (STR) models have been developed to take care of these regimes by modelling the transition as a continuous process dependent on the transition variable which allows for incorporating regime switching behaviours. Terasvirta’s (1994) proposed a nonlinear Smooth Transition Autoregressive (STAR) model, which classifies financial markets into two phases of recession and expansion. The model gives a continuous time series movement between two discrete states, 0 and 1, determined by the transition functions. STR has been extensively used to study exchange rates and has recently been applied to Phillips curve. Also, the methodology has been extended recently to panel data which allows for a whole spectrum of new applications in modelling several variables and incorporating heterogeneity in disaggregated data.

The daily closing prices of the Nigerian stocks from January 1996 to December 2011 were examined by Emenike and Aleke (2012) using asymmetric GARCH variants. Their result showed strong evidence of asymmetric effects in the stock returns and therefore proposed EGARCH as performing better than other asymmetric rivals. The forecasting properties of linear GARCH model for daily closing stocks prices of Zenith bank Plc in the Nigerian Stock Exchange was also studied in Arowolo (2013). The Akaike and Bayesian Information Criteria (AIC and BIC) techniques were used to obtain the order of the GARCH (p,q) that best fit the Zenith Bank return series. The information criteria identified GARCH (1,2) as the appropriate model. His result further supported the claim that financial data are leptokurtic.

Since the great depression of the financial global crisis, the world, particularly the developing countries, are currently experiencing one of the worst bear markets. The market phase is characterize by the bull and bear markets which corresponds to periods of generally increasing and decreasing market prices respectively, and recent research has shown that bull markets persist longer than bear markets (Gil-Alana et al., 2014). In this interest, Yaya and Gil-Alana (2014) examined persistence and asymmetric volatility in the Nigerian stock bull and bear markets. They employed estimate of the fractional difference parameter as a stability measure of the degree of persistence in the level of the series and in the squared returns. Their results showed that the level of persistence differ between the two market phases in both level and squared return series.

3.0 Methodology

3.1 The Linear and Nonlinear GARCH specifications

The initial proposition of ARCH and GARCH models by Engle (1982) and Bollerslev (1986), respectively was the linear ARCH[5] specifications upon which the nonlinearity test was built (Hagerud, 1996 and Gonzalez-Rivera, 1998). The $ARCH(q)$ specification is defined as,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (1)$$

where $\varepsilon_t = z_t \sigma_t$ with $z_t \approx nid(0,1)$, that is standard normal variate, and t is the time factor. The α_0 and α_1 are the constant and first order ARCH parameter, q is the number of autoregressive lag, $\varepsilon_t = r_t$, and r_t series is the transformed log-returns of difference of prices, P_t and σ_t is the conditional standard deviation series. Actually, Engle's (1982) proposition ensured positivity of conditional standard deviation series as well as stationarity once $\alpha_0 \geq 0$ and $\alpha_1 < 1$. The $GARCH(p,q)$ specifications is,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \quad (2)$$

where q, p are the number of lags for ARCH and GARCH terms, and the lags of the conditional variance, σ_{t-i}^2 represents the regressors in the model. The GARCH model ensures positivity and stationarity of conditional variance series with the conditions $\alpha_0 \geq 0$ and $\alpha_1 + \beta_1 < 1$ .Whenever $\alpha_1 + \beta_1 = 1$, the model realized nonstationary conditional variances and hence, the model is termed Integrated GARCH (IGARCH) model of Engle and Bollerslev (1986).

The ST-ARCH(q) model proposed by Hagerud is given as;

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_{1j} \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \alpha_{2j} \varepsilon_{t-j}^2 G\left(\varepsilon_{t-j}\right) \quad (3)$$

where $G(.)$ are the transition functions, with form,

[5]Actually, the ARCH and GARCH models are types of nonlinear time series models, but within the class of GARCH variants, they belong to linear types of GARCH models since there are nonlinear types of GARCH models in the literature (See Nelson (1991), Glosten et al. (1993), Ding et al (1993), Hagerud (1996, 1997) and Gonzalez-Rivera (1998)).

$$G\left(\varepsilon_{t-j}\right)=\left(1+\exp\left[-\theta\varepsilon_{t-j}\right]\right)^{-1}-\frac{1}{2}, \quad \theta>0 \tag{4}$$

for the LST-ARCH model, and

$$G\left(\varepsilon_{t-j}\right)=1-\exp\left(-\theta\varepsilon_{t-j}^{2}\right), \quad \theta>0 \tag{5}$$

for the EST-ARCH model. From (3), the parameters of the model are $\alpha_0,\alpha_{1j},\alpha_{2j}\left(j=1,...,q\right)$. From the transition functions in (4) and (5), θ is the threshold parameter which causes the nonlinearity, and the transition variable is ε_{t-j}, this is actually ε_{t-1} as applied in this work.[6]For the LST-ARCH model (3 with 4), stationarity of the return process is ensured by $\sum_{j=1}^{q}\left(\alpha_{1j}-\frac{1}{2}\left|\alpha_{2j}\right|+\max\left(\alpha_{2j},0\right)\right)<1$, and the sufficient conditions for strictly positive conditional variance σ_t^2 is ensured by setting $\alpha_0>0,\alpha_{1j}\geq 0$ and $\alpha_{1j}\geq\frac{1}{2}\left|\alpha_{2j}\right|$. For the EST-ARCH model (3 with 5), stationarity of the process is ensured by $\sum_{j=1}^{q}\left(\alpha_{1j}+\max\left(\alpha_{2j},0\right)\right)<1$, while the sufficient conditions for strictly positive conditional variance σ_t^2 is ensured by setting $\alpha_0>0,\alpha_{1j}\geq 0$ and $\alpha_{1j}+\alpha_{2j}\geq 0$ (see MilhØj, 1985) and TjØstheim, 1986).

The generalized version of $ST-ARCH\left(p,q\right)$model which is defined as the $ST-GARCH\left(p,q\right)$ is given in Gonzalez-Rivera (1998) as,

$$\sigma_t^2=\alpha_0+\sum_{j=1}^{q}\alpha_{1j}\varepsilon_{t-j}^2+\sum_{j=1}^{q}\alpha_{2j}\varepsilon_{t-j}^2G\left(\varepsilon_{t-j}\right)+\sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \tag{6}$$

with the addition of the lagged conditional variances in the equation as in the $GARCH\left(p,q\right)$ model of Bollerslev (1986). Then, for positive conditional variance in the LST-GARCH model, it is required that $\alpha_0>0,\alpha_{1j}\geq 0,\beta_k\geq 0\left(j=1,...,q\right)$ and $\alpha_{1j}\geq\frac{1}{2}\left|\alpha_{2j}\right|$, and for stationarity of the return process, it is required that $\sum_{j=1}^{q}\left(\alpha_{1j}-\frac{1}{2}\left|\alpha_{2j}\right|+\max\left(\alpha_{2j},0\right)\right)+\sum_{j=1}^{p}\beta_j<1$

For the positive conditional variance in the EST-GARCH model, it is required that $\alpha_0>0,\alpha_{1j}\geq 0,\beta_j\geq 0\left(j=1,...,p\right)$ and $\alpha_{1j}+\alpha_{2j}\geq 0$, and for stationarity of

[6]See van Dijk et al. (2002) for different forms that transition variable may assume.

the process, $\sum_{j=1}^{q}\left(\alpha_{1j}+\max\left(\alpha_{2j},0\right)\right)+\sum_{j=1}^{p}\beta_{j}<1$. Clearly, for nonlinear smooth transition to be defined in both ST-ARCH and ST-GARCH models, it is required that at least one $\alpha_{ij}>0\ \left(i=1,2\right)$.

The ST-GARCH model in (6) is a flexible nonlinear GARCH model where the idea behind STAR modelling, in the conditional mean, is adopted to the nonlinear conditional volatility specifications. The conditional variance of LST-GARCH model possesses dynamics similar to those of the GJR-GARCH model in that the parameter for the squared returns have one value when the returns is positive, and another when the returns is negative. However, in the exponential smooth transition ARCH model, the dynamics of the conditional variance are independent of the sign of lagged returns. Instead, the magnitudes of lagged squared returns control the conditional variance. This specification is similar to that proposed in Engle and Bollerslev (1986), only that the transition function is not a cumulative distribution function but instead, it is the exponential function, which means that specification tests are easier to derive.

3.2 Data Source, Transformation and Test Procedures

The data used in this work are the daily share prices of 12 highly capitalized banks listed on the platform of the Nigerian Stock Exchange (NSE) spanning from 4th January, 2007 to 2ndApril, 2015. The stock market index constitutes daily equity trading of all listed and quoted companies in the Nigeria Stock Exchange. The sample period was based on data availability from the NSE and this period covers the time of global financial crisis in Nigeria (2008). The data were sourced from the Capital Assets website (www.capitalassets.com.ng). Since the outcome of this paper will be so sensitive to monetary agency and banks concerned, we therefore reshuffled the list of banks, and renamed them: Bank 1, Bank 2 up to Bank 12, hence we refer to them as these new names in the sub-sequent parts of the paper.

Since emphasis is on the volatility in the returns series, we therefore transformed each bank share daily price series to log-return series as follows: let P_t represent the daily share closing price at day *t*; then at previous day *t-1*, we had P_{t-1}. The log-return series is then computed by taking the first price difference of the logarithms, that is, $r_t = \log(p_t) - \log(p_{t-1})$ and the squared log-returns are computed as r_t^2.

3.3 Long Range Dependence technique

To compute the estimate of Long Range Dependence (LRD),*d*, we employ the local Whittle estimator which is often presented in the frequency domain,

$$f(\lambda) \sim G|\lambda|^{-2d}, \quad \text{as } \lambda \to 0 \tag{7}$$

where G is a constant. The computation requires additional parameter m and sample size N such that $m < N/2$, and as $N \to \infty$, $1/m + m/N \to 0$, that is, as the size of N increases, m also increases, although at slower rate. The log-likelihood of the spectral density in (7) is given as,

$$L\left\{I(\lambda_j),...,I(\lambda_m),\theta\right\} = \prod_{j=1}^{m} \frac{1}{G_\theta(\lambda_j)} e^{-I(\lambda_j)G(\lambda_j)^{-1}} \tag{8}$$

which is minimized by the likelihood function

$$Q(G,d) = \sum_{j=1}^{m} \left\{-\log\left(G\lambda_j^{-2d}\right) - I(\lambda_j)G\lambda_j^{2d}\right\} . \tag{9}$$

where $\lambda_j = 2\pi j/N$ and $I(\lambda_j)$ is the periodogram $I(\lambda_j) = \frac{1}{2\pi N}\left|\sum_{t=1}^{N} r_t^2 e^{i\lambda_j t}\right|^2$ for the squared log-returns time series r_t^2. Replacing the above function G by its estimate $\hat{G}$,

$$\hat{G} = m^{-1}\sum_{j=1}^{m}\left\{I(\lambda_j)\lambda_j^{2d}\right\} \tag{10}$$

Then, putting (10) in (9), the local Whittle estimate of d is obtained by minimizing the residual estimates from the likelihood,[7]

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[7] Robinson (1995) showed that the estimator is consistent for $d \in (-0.5, 0.5)$ and this consistency depends on the value set for m.

$$R(d) = Q(\hat{G}, d) = \log\left\{ m^{-1}\sum_{j=1}^{m} I(\lambda_j)\lambda_j^{2d} - 2m^{-1}d\sum_{j=1}^{m}\log(\lambda_j)\right\} \quad (11)$$

The significance of volatility persistence measure, d is then obtained based on the Wald statistic given by,

$$W = \left(\frac{d}{s}\right)^2 \quad (12)$$

where s is the standard error associated with d. This statistic is χ_1^2 distributed.

3.4 Linearity and Specification Test

Following Hagerud (1996), we present the procedures for testing the null of linear conditional variance against the alternative of non-linear conditional variance. This involves testing the null of no ARCH effect in the standardized errors against nonlinear ST-ARCH. Conditional homoscedasticity of returns against ST-ARCH model is investigated by re-specifying the model in two-regime as,

$$\sigma_t^2 = \omega_{10} + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2\left[1 - F(\varepsilon_{t-i})\right] + \sum_{j=1}^{p}\gamma_j\varepsilon_{t-j}^2 F(\varepsilon_{t-i}) \quad (13)$$

As in the STAR model of Teräsvirta (1994), nonlinearity in (3) is tested based on the null hypothesis,

$$H_0 : \alpha_{1j} = \alpha_{2j} = 0 (j = 1,...,q) \quad (14)$$

against the alternative; H_1: at least one $\alpha_{ij} \neq 0$, where θ is from the transition functions (4) and (5). The expectation is that, there should be no evidence to accept the null for GARCH model to be applicable. Since this parameter is not identified under the null hypothesis, the transition function is then approximated by a lower order Taylor series approximation (see Luukkonnen et al., 1988). Firstly, approximating the Logistic Smooth Transition (LST) function in (4) by the first order Taylor's series expansion given by;

$$G(\varepsilon_{t-j}) \cong G(\varepsilon_{t-j})_{|\theta=0} + \theta\frac{\partial G(\varepsilon_{t-j})}{\partial\theta 1!}_{|\theta=0} \quad (15)$$

where,

(a) $$G(\varepsilon_{t-j}) = \left(1+\exp\left[-\theta\varepsilon_{t-j}\right]\right)^{-1} - \frac{1}{2} \text{ and } G(\varepsilon_{t-j})_{|\theta=0} = 0;$$

(b) $$\frac{\partial G(\varepsilon_{t-j})}{\partial\theta 1!} = \frac{\varepsilon_{t-j}\exp(-\theta\varepsilon_{t-j})}{\left[1+\exp\left[-\theta(\varepsilon_{t-j})\right]\right]^2} \text{ and } \frac{\partial G(\varepsilon_{t-j})}{\partial\theta 1!}_{|\theta=0} = \frac{1}{4}\varepsilon_{t-j}$$

Then simplifications in (a) and (b) can be substituted in (15) to obtain the resulting approximation for the LST function as;

$$G'(\varepsilon_{t-j})_{|\theta=0} \approx \frac{1}{4}\theta\varepsilon_{t-j} \quad (16)$$

Substituting (16) above in the ST-ARCH model in (3) results in the auxiliary regression model,

$$\hat{\sigma}_t^2 = \alpha_0^* + \sum_{j=1}^{q}\alpha_{1j}^*\varepsilon_{t-j}^2 + \sum_{j=1}^{q}\alpha_{2j}^*\varepsilon_{t-j}^3 \quad (17)$$

so that the null hypothesis in (14) is then equivalent to testing,

$$H_0 : \alpha_{21}^* = ... = \alpha_{2q}^* = 0$$

The specification procedure involves computing the residual sum of squares; $SSR_0 = \sum_{t=1}^{N}\hat{\varepsilon}_t^2$ from the linear GARCH model and regressing the squared residuals on the vector $\hat{\boldsymbol{\omega}} = \left\{1, \varepsilon_{t-1}^2, ..., \varepsilon_{t-q}^2, \varepsilon_{t-1}^3, ..., \varepsilon_{t-q}^3\right\}$, with the residual sum of squares denoted as SSR_1. Hence, the Lagrange Multiplier (LM) test statistic is computed as;

$$LM_1 = \frac{SSR_0 - SSR_1}{SSR_0/N} = NR^2 \quad (18)$$

where R^2 is the coefficient of multiple determination and N is the sample size. The test statistic is distributed as χ^2 distribution with 2p+1 degrees of freedom.

Furthermore, approximating the exponential function in (5) by the first order Taylor’s series expansion:

$$G(\varepsilon_{t-j}) \cong G(\varepsilon_{t-j})_{|\theta=0} + \theta\frac{\partial G(\varepsilon_{t-j})}{\partial\theta 1!}_{|\theta=0} \quad (19)$$

where;

(c) $$G(\varepsilon_{t-j}) = 1 - \exp(-\theta\varepsilon_{t-j}^2) \text{ and } G(\varepsilon_{t-j})_{|\theta=0} = 0$$

(d) $$\frac{\partial G(\varepsilon_{t-j})}{\partial\theta 1!} = \varepsilon_{t-j}^{2}\exp(-\theta\varepsilon_{t-j}^{2}) \text{ and } \frac{\partial G(\varepsilon_{t-j})}{\partial\theta 1!}_{|\theta=0} = \varepsilon_{t-j}^{2}$$

Then (c) and (d) can be substituted in (19) to get the resulting approximation for the exponential function as;

$$G^{'}(\varepsilon_{t-j})_{|\theta=0} \approx \theta\varepsilon_{t-j}^{2} \tag{20}$$

then, substituting (20) above in (3) gives the auxiliary regression model as;

$$\hat{\sigma}_t^2 = \alpha_0^* + \sum_{j=1}^{q}\alpha_{1j}^*\varepsilon_{t-j}^2 + \sum_{j=1}^{q}\alpha_{3j}^*\varepsilon_{t-j}^4 \tag{21}$$

We then test the null hypothesis,

$$H_0 : \alpha_{31}^* = ... = \alpha_{3q}^* = 0$$

with SSR_0 computed in similar manner as that of LST-ARCH above, the sum of squares residual (SSR_2) is then obtained by regressing $\hat{\varepsilon}_t^2$ on $\hat{\boldsymbol{\omega}} = \left\{1, \varepsilon_{t-1}^2, ..., \varepsilon_{t-q}^2, \varepsilon_{t-1}^4, ..., \varepsilon_{t-q}^4\right\}$ as above and the LM test statistic computed as;

$$LM_2 = \frac{SSR_0 - SSR_2}{SSR_0/N} = NR^2 \tag{22}$$

which is also distributed as χ^2 distribution with 2p+1 degrees of freedom.

The null hypothesis of linear ARCH can alternatively be tested against the alternative of nonlinear ARCH for both LST and EST functions simultaneously by using the auxiliary regression model, obtained by combining the two regressions in (17) and (21). This becomes,

$$\hat{\sigma}_t^2 = \alpha_o^* + \sum_{j=1}^{q}\alpha_{1j}^*\varepsilon_{t-j}^2 + \sum_{j=1}^{q}\alpha_{2j}^*\varepsilon_{t-j}^3 + \sum_{k=1}^{q}\alpha_{3k}^*\varepsilon_{t-k}^4 \tag{23}$$

An LM type test statistics for the hypothesis is then computed as $LM_3 = NR^2$ which is distributed as χ^2 distribution with 3p+1 degrees of freedom. A Closer look at the regression in (23) indicates that the parameters α_{2j}^* and α_{3k}^* are contributed individually by LST and EST functions. Therefore, the decision on the specification is that, if the rejection probabilities of estimates α_{2j}^* are stronger at specified level of significance, than those of α_{3k}^*, we then select LST specification, otherwise we select EST specification.[8]

[8]Testing homoscedasticity against heteroscedasticity is achieved via classical ARCH test. The linearity against nonlinearity of ARCH test is based on the ARCH test for heteroscedasticity. Therefore, the auxiliary regressions from ST-ARCH models are sufficient for testing for

4.0 The Empirical Results

The plots of the return series for the share price are presented in Figure 1. These show periods of low and high volatilities, which signify volatility clustering. The log-return series plots for Banks 10 and 11 showed some period of calmness as displayed on the plots.

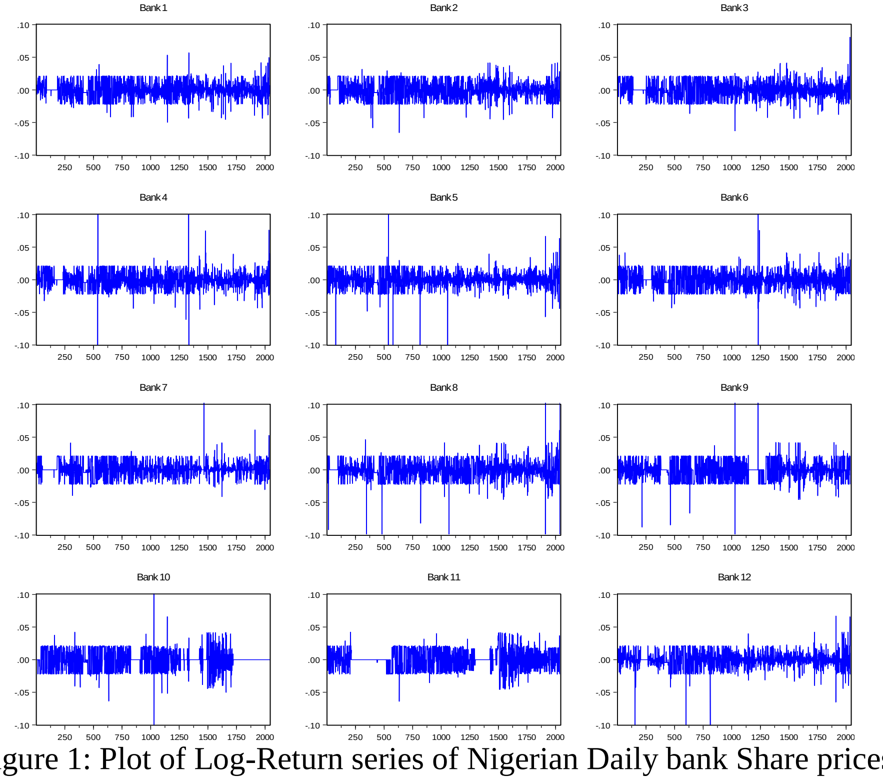

Figure 1: Plot of Log-Return series of Nigerian Daily bank Share prices

Since we do not observe clear volatility persistence among the bank share prices using plots of log-returns, we then estimated volatility persistence based on the estimate of the long range dependence of the squared log-returns. Significant volatility persistence measure was observed in ten (10) bank share prices with Bank 6 exhibiting the highest volatility persistence and Bank 12 exhibiting the lowest volatility, as shown in Table 1.

nonlinear ARCH in the context of this work. It is very straight forward. Interested readers can base their tests on ST-GARCH auxiliary regression and obtain similar LM test with different degrees of freedom as a result of extra parameter in the models (see Hagerud, 1997).

Table 1: Estimates of Volatility persistence for bank share prices

Banks	*d*	*S*	*W*
1	0.2053***	0.0156	173.19
2	0.2124***	0.0156	185.38
3	0.2016***	0.0156	167.01
4	0.4271***	0.0156	749.57
5	0.1478***	0.0156	89.76
6	0.4344***	0.0156	775.41
7	0.0259	0.0156	2.76
8	0.3889***	0.0156	621.48
9	-0.0006	0.0156	0
10	0.3309***	0.0156	449.93
11	0.2831***	0.0156	329.33
12	0.0513***	0.0156	10.81

Source: *Computed and compiled by the authors*
Note: *The d is the long range dependence estimate), s is the standard error of the estimate, W is the Wald statistic which is chi-square distributed with 1 degree of freedom. *** represent statistical significance at 1% level*

Since the conventional GARCH model failed to capture the asymmetric properties of the banks series, we introduced estimation of nonlinear GARCH type's models (ST-ARCH and ST-GARCH) proposed by Hagerud (1996) and Gonzalez-Rivera (1998) for capturing asymmetric and symmetric nonlinearity of conditional variance of the bank series. In order to determine the appropriate nonlinear GARCH type model for modelling each of the bank series, the linearity and specification test was first performed on the return series for each bank. Tables 2a and 2b below reports the test of conditional homoscedasticity of returns against the alternative ST-ARCH model for the logistic (LM_1) and exponential (LM_2) transition function and also a test of null of conditional homoscedasticity against the alternative of ST-ARCH for both types of smooth transition simultaneously (LM_3).

Table 2a: Linearity and Specification tests based on LM1 and LM2 tests

Banks	LM_1				LM_2				Decision
	R^2	NR^2	$\hat{\alpha}^*_{21}$	p-value	R^2	NR^2	$\hat{\alpha}^*_{31}$	p-value	
1	0.0244	49.9309	-2.5603	0.0049	0.0257	52.4234	-120.804	0.0012	EST-ARCH
2	0.0268	54.7115	0.6724	0.4302	0.0356	72.7104	-111.401	0	EST-ARCH
3	0.0116	23.6171	-0.2177	0.7372	0.0173	35.2622	-44.9016	0.0006	EST-ARCH
4	0.776	1585.4497	-6.5961	0	0.4402	899.2877	37.8035	0	LST-ARCH
5	0.1471	300.4232	-3.803	0	0.1209	246.9578	36.904	0.0001	LST-ARCH
6	0.0323	65.8868	0.3939	0	0.024	49.0116	-5.6001	0.0005	LST-ARCH
7	0.0083	16.9773	-1.7437	0.0001	0.0071	14.5053	-8.9414	0.0002	LST-ARCH
8	0.0001	0.2247	-0.0177	0.847	0.0009	1.9204	-1.5857	0.1875	Linear-ARCH
9	1.00E-06	0.002	0.0104	0.99	0	0.002	-0.0393	0.9753	Linear ARCH
10	0.1439	294.0694	2.5647	0	0.0811	165.7077	-16.382	0.0001	LST-ARCH
11	0.0932	190.428	0.4046	0.6484	0.0944	192.8183	-62.0977	0.0921	Linear-ARCH
12	0.0029	5.8287	1.2629	0.0288	0.0035	7.1914	-8.1367	0.0132	EST-ARCH

Source: *Computed and compiled by the authors*
Note:*LM_1 and LM_2 represent the Lagrange Multiplier for the Logistic and exponential ARCH, respectively and are both chi-square distributed with (2p+1) degrees of freedom. R^2 is the coefficient of multiple regression. $\hat{\alpha}^*_{21}$ and $\hat{\alpha}^*_{31}$ are the parameters for LST-ARCH and EST-ARCH, respectively.*

Table 2b: Linearity and Specification tests based on LM3 test

Banks	LM_3						Decision
	R^2	NR^2	$\hat{\alpha}^*_{21}$	p-value	$\hat{\alpha}^*_{31}$	p-value	
1	0.0278	56.6933	-1.95819	0.0364	-101.143	0.0085	EST-ARCH
2	0.0382	78.0222	-2.49688	0.0191	-156.631	0	EST-ARCH
3	0.0192	39.2256	1.58449	0.0447	-63.1694	0.0001	EST-ARCH
4	0.8278	1691.0933	-6.6764	0	41.5513	0	LST-ARCH
5	0.1482	302.8543	-3.60414	0	16.2529	0.092	LST-ARCH
6	0.0367	74.9577	0.37518	0	-4.90239	0.0022	LST-ARCH
7	0.009	18.4279	-4.32627	0.047	14.6887	0.2262	LST-ARCH
8	0.001	1.9613	0.01897	0.8428	-1.65809	0.1873	Linear-ARCH
9	0.0001	0.1022	1.74785	0.7628	-2.68157	0.7618	Linear ARCH
10	0.144	294.0899	2.58071	0	0.51593	0.8665	LST-ARCH
11	0.0944	192.8592	-0.17502	0.8547	-1.63236	0.1028	Linear-ARCH
12	0.0035	7.2322	-0.281	0.8441	-9.5985	0.2375	EST-ARCH

Source: *Computed and compiled by the authors*
Note:*LM_3 represents the Lagrange Multiplier for the combination of both Logistic and exponential ARCH and is chi-square distributed with (3p+1) degrees of freedom. R^2 is the coefficient of multiple regression. $\hat{\alpha}^*_{21}$ and $\hat{\alpha}^*_{31}$ are the parameters for LST-ARCH and EST-ARCH, respectively.*

The result shows that the daily price returns series of Bank 8, Bank 9 and Bank 11 display linear GARCH specifications while the remaining banks' return series follow nonlinear smooth transition volatility. Bank 1, Bank 2, Bank 3 and Bank 12 follow the Exponential Smooth Transition type, while Bank 4, Bank 5, Bank 6, Bank 7 and Bank 10 follow the Logistic Smooth Transition type. The summary of the linear and nonlinear volatility models for the returns series of daily price for all the banks is given in Tables 3a and 3b below.

Table 3a: Estimated Linear and Nonlinear Volatility models for Banks 1 – 8

	Bank 1			Bank 2		
	GARCH(1,1)	EST-ARCH(1,1)	EST-GARCH(1,1)	GARCH(1,1)	EST-ARCH(1,1)	EST-GARCH(1,1)
α_0	5.26E-06	0.0001	0.0001	6.77E-06	8.91E-05	8.91E-05
α_{11}	0.1408	0.3755	0.3393	0.1934	0.7402	0.7286
$\hat{\alpha}_{21}$		0.1318	0.1167		-0.6589	-0.658
$\hat{\beta}_1$	0.8329	-NA-	0.0316	0.7782	-NA-	0.0114
$\hat{\theta}$		436.7238	454.1857		1278.623	1278.272
LogL	6222.713	12936.62	12937.11	6146.55	12991.3	12994.38
SSE	0.3448	0.0004	0.0004	0.3448	0.0004	0.0004
AIC	-6.0888	-12.6666	-12.6661	-6.0142	-12.7231	-12.7222
SIC	-6.0805	-12.6556	-12.6524	-6.006	-12.7121	-12.7084
	Bank 3			Bank 4		
	GARCH(1,1)	EST-ARCH(1,1)	EST-GARCH(1,1)	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)
α_0	6.84E-06	9.75E-05	9.76E-05	1.26E-05	0.0002	0.0002
α_{11}	0.2313	0.6255	0.6181	0.3982	0.0305	0.1537
$\hat{\alpha}_{21}$		-0.5903	-0.5844		1.81E-05	9.75E-06
$\hat{\beta}_1$	0.7486	-NA-	0.0044	0.6276	-NA-	-0.0399
$\hat{\theta}$		1457.466	1457.701		47.3274	49.6662
LogL	6330.094	13033.17	13033.18	6167.179	8144.818	8542.984
SSE	0.3181	0.000342	0.000342	0.4535	0.041	0.0278
AIC	-6.096	-12.76118	-12.76022	-6.0344	-7.9734	-8.3624
SIC	-6.0878	-12.75017	-12.74645	-6.0262	-7.9624	-8.3486
	Bank 5			Bank 6		
	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)
α_0	9.00E-06	0.0001	0.0001	1.26E-05	0.0001	9.23E-05
α_{11}	0.2454	-0.1838	-64.3658	0.2246	0.1411	-0.2328
$\hat{\alpha}_{21}$		0.2611	42.5913		-1.76E-05	0.8116
$\hat{\beta}_1$	0.7522		0.4313	0.7205		0.0744
$\hat{\theta}$		10.5555	0.1939		53.2705	-3.04E-06
LogL	6216.329	10357.98	10361.45	6046.41	12664.53	10361.45
SSE	0.3416	0.0047	0.0047	0.4138	0.0005	0.0047
AIC	-6.0826	-10.141	-10.1434	-5.9162	-12.4001	-10.1434
SIC	-6.0743	-10.13	-10.1297	-5.908	-12.3891	-10.1297
	Bank 7			Bank 8		
	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)
α_0	5.43E-05	0.0001	0.0001	5.66E-06		
α_{11}	0.2635	-0.2183	-0.5644	0.2968		
$\hat{\alpha}_{21}$		0.3422	0.4799			
$\hat{\beta}_1$	0.3729		0.1231	0.7737		
$\hat{\theta}$		11.7712	1.85E-08			
LogL	6255.762	12035.59	12038.5	5793.067		
SSE	0.2876	0.0009	0.0009	0.6589		
AIC	-6.1212	-11.7841	-11.786	-5.6682		
SIC	-6.1181	-11.7731	-11.7722	-5.6599		

Source: *Computed and compiled by the authors*

Table 3b: Estimated Linear and Nonlinear Volatility models for Banks 9 -12

	Bank 9			Bank 10		
	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)	GARCH(1,1)	LST-ARCH(1,1)	LST-GARCH(1,1)
α_0	0.0003			1.27E-06	0.0001	-0.0812
α_{11}	0.0014			0.3722	29.3156	-0.3945
$\hat{\alpha}_{21}$					-19.2989	0.6498
$\hat{\beta}_1$	0.5711			0.6907		0.0246
$\hat{\theta}$					0.1326	0.0542
LogL	4875.855			6586.337	12112.8	12113.14
SSE	0.929			0.4732	0.0008	0.0008
AIC	-4.7703			-6.4448	-11.8597	-11.8591
SIC	-4.762			-6.4365	-11.8487	-11.8453
	Bank 11			Bank 12		
	GARCH(1,1)			GARCH(1,1)	EST-ARCH(1,1)	EST-GARCH(1,1)
α_0	3.99E-07			4.41E-06	9.79E-05	8.59E-05
α_{11}	0.1851			0.2169	0.6543	0.907
$\hat{\alpha}_{21}$					-0.659	-0.9993
$\hat{\beta}_1$	0.8285			0.7918	-NA-	0.0953
$\hat{\theta}$					307.6499	984.6778
LogL	6584.101			6379.2	10674.69	10679.45
SSE	0.4251			0.3114	0.0034	0.0034
AIC	-6.4426			-6.242	-10.4512	-10.4549
SIC	-6.4343			-6.2337	-10.4402	-10.4411

Source: *Computed and compiled by the authors*

Tables 3a and 3b show that the volatility of the returns series of Bank 8, Bank 9 and Bank 11 are adequately captured by the GARCH(1,1) model with higher volatility persistence observed for Bank 8 and Bank 11, while the volatility experienced by Bank 9 is of lower persistence. The ARCH parameter, α_{11} for Bank 9 is relatively low in comparison to Bank 8 and Bank 11, which implies that, while volatility of Bank 9 does not react intensely to market movement, it does for Bank 8 and Bank 11. By implication, the conditional variance will take a long time to restore to steady state. The relatively large GARCH lag coefficient $\hat{\beta}_1$ reveals volatility persistence for the three banks. In Tables 3a and 3b, the remaining banks' volatility are more adequately capture by the non-linear GARCH type models.

Specifically, the selection of the best model, which is based on the information criteria (AIC and SIC), shows that the volatilities of Bank 1, Bank 2, Bank 3 and Bank 7 are adequately captured by EST-ARCH(1,1) model, Bank 4 and Bank 5 are well captured by LST-GARCH(1,1), Bank 6 and Bank 10 are both captured by LST-ARCH(1,1), while Bank 12 is explained by EST-GARCH(1,1). The GARCH(1,1) model does not out-perform the non-linear GARCH type models since it was unable to account for non-linearity observed in the return series.

5.0 Concluding Remarks

In this work, we suggested possibility of linear and nonlinear volatility model specifications for the daily closing share prices of twelve (12) highly capitalized banks in the Nigerian Stock Exchange (NSE). Based on long range dependence approach on the squared log-returns series, Bank 6 was identified with the highest volatility persistence while Bank 12 was identified with the least volatility persistence. The volatility of the Nigerian bank share prices is further confirmed with three (3) of the banks found to exhibit linear volatility behaviour, while the remaining nine (9) revealed nonlinearity characteristics.

High volatility persistence in the financial market will have a direct high impact consequence on portfolios. On the part of the investors, it adds to their worries as they keep watch on market values of their portfolios. The banking sector contributes a higher percentage on the overall capital markets, and so bank share volatility has the highest effects on the overall volatility persistence of the financial market. Many banks in Nigeria have been closed down or merged or even taken over by the central bank because of the issues of insolvency and liquidity caused by non-performing loans. This is essentially due to the fact that, banks stocks in Nigeria has experienced high rate of volatility clustering over time.

As observed in this study, almost all the banks reacted intensely to market price movement, and the implication of this is that investors, policy makers and banks in Nigeria needs to focussed on risk-adjusted returns, risk parity, and volatility targeting strategies. An understanding of the GARCH-type behaviour (linear or nonlinear) for each banks' share price will provide a robust framework for the process of risk budgeting, especially in the present state of the Nigerian economy. While, the impact of news cannot, and should not be ignored in the process of making expectations on investments, relevant agencies should understand the volatility behaviour of banks share prices in order to be guided on how to address the underlying issues in the Nigerian banking system.

In order to further establish the linearity and/or nonlinearity of the daily bank share prices, in a similar fashion, we could consider the Generalized Autoregressive Score (GAS) model and Asymmetric Power ARCH (APARCH) models in modelling volatility in Nigerian banks' share prices, and check if the suggested linear or nonlinear ARCH/GARCH models yield better prediction when compared with nonlinear GAS and APARCH models.

Another promising research interest is to allow smooth transition in GAS and APARCH models as in ST-GARCH model.

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